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Analysis of the Design Calculations for Electrical Earthing Systems

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Abstract – *Earthing systems play an essential role in electrical systems in terms of safety for people in the vicinity against the hazard of electric shocks as well as protection and proper operation of equipment during the incidence of faults. Both are achieved by providing a low-impedance path that can dissipate fault currents to the conductive mass of Earth. One of the first steps in the design of an earthing system is estimating the total resistance to earth and determining the proper size and basic layout of the earth electrode required. Engineers must design adequate earthing systems that comply with international standards and national regulations, which in fact incorporate a variety of earthing methods and various formulae to obtain the design parameters, especially the earthing resistance. The effects of using the varied formulae for each earthing technique are ambiguous and entail considerable risks, and thus require comprehensive compilations and critical examinations. This paper reviews many of the earthing design formulae available in widely used international standards and published scientific papers for a comparative analysis of their differences. The results of a rigorous survey for each earthing type, based on respective electrical resistances' calculations evaluated for a specified range of resistivities, are presented in line graphs to show precise trends. A recommended list of the most conservative formulae from a safety perspective, based on the results obtained, is outlined as a basis for computing the earthing resistance for designing effective earthing systems. This provides a beneficial compact reference to facilitate the revision and provision of international earthing standards agreement.* ***Copyright © 2021 The Authors.***

***Keywords**: Earthing System, Electrical Safety Standards, Resistance Calculations, Soil Resistivity*

Nomenclature

α	Improvement constant
β	Improvement constant
δ	Thickness of concrete between rods and soil [m]
λ	Factor given in Table 1 of BS-7430 [2]
ρ	Soil resistivity assumed uniform [Ω m]
ρ_c	Resistivity of concrete or conductive backfill [Ω m]
a	Radius of each rod strip wire or round plate [m]
A	Area of one face of the plate or one mesh grid [m^2]
b	Length of the long side of the grid [m]
d	Diameter of each rod or wire [m]
D	Diameter of ring [m]
D_c	Diameter of the concrete shell [m]
h	Burial depth of electrode [m]
k_1	Coefficient of the grid resistance equation
k_2	Coefficient of the grid resistance equation
L	Total length of buried conductor rod (electrode) [m]
L_c	Total length of all connected grid conductors [m]
L_n	Nominal length of each electrode [m]
L_r	Total length of the ground rod [m]
L_T	Total length of buried conductors used in grid [m]
n	Number of vertically driven rods (electrodes)
N	Total number of joint vertical electrodes
P	Coefficients given in Table 6 of BS-7430 [2]
Q	Coefficients given in Table 6 of BS-7430 [2]
R	Resistance of the hemispheric ground [Ω]
r_1	Resistivity of the upper layer [Ω cm]
r_2	Resistivity of the subsoil [Ω cm]
R_1	Resistance of a single strip or one footing [Ω]
R_{2r}	Resistance of 2 ground rods arranged vertically [Ω]
R_{3s}	Resistance of a star arranged strip [Ω]
R_{4s}	Resistance of 4 strips set in a cruciform [Ω]
R_a	Resistance of rods in a hollow square [Ω]
R_b	Resistance of the backfilled electrode [Ω]
R_q	Resistance of the grounding grid [Ω]
R_h	Resistance of the buried horizontal rod [Ω]
R_{hp}	Resistance of the horizontal round plate [Ω]
R_{hs}	Resistance of the horizontal strip electrode [Ω]
R_{hw}	Resistance of the buried horizontal wire [Ω]
R_L	Resistance of the right-angle turn of wire [Ω]
R_m	Resistance of the mesh or grid [Ω]
R_n	Resistance of n conductors in parallel [Ω]
R_o	Low current resistance of vertical grounding rod [Ω]
r_o	Rod radius [m]
R_p	Resistance of plate to ground [Ω]

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R_r Resistance of rod or ring of wire electrode [Ω]
R_s Resistance of rod strip or round conductor [Ω]
R_t Total resistance to ground [Ω]
R_{TOT} Total resistance of hollow square electrodes [Ω]
R_v Resistance of the vertical earth rod [Ω]
R_{vp} Resistance of the vertical round plate [Ω]
s Spacing between two adjacent rods [m]
$s/2$ Depth of burial of electrode [m]
w Width of strip [m]

I. Introduction

Earthing of either electrical equipment or a whole system requires the provision of a connection to the thin layered conductive mass material that covers the surface of Earth. The total resistance of this connection should not be greater than the required resistance value as stipulated in international standards, such as IEEE [1], in order to operate the safety mechanism installed in the system, which isolates the electricity supply from overcurrent. This connection should also be capable of carrying the maximum expected fault current [2]. There are several ways of connecting an electrical installation to the earth, including the use of different earthing electrodes such as rods, tapes, and plates. The most popular method by far is using rod electrodes driven deep into a conductive ground. Similarly, plate electrode earthing must be buried at a sufficient depth, which entails considerable excavation since the plates are typically 1 or 2 m^2. The tape earthing type is mostly utilised in large power substations, where the earthing tape is arranged in a mesh form and laid in trenches over the whole site. This mesh is then used for earthing the entire substation. If individual earthing electrodes are not installed properly, or if the spacing between them is not physically sufficient, the overall effectiveness of the earthing system is severely impaired [3]. The choice of an earthing system depends on both the type of installation and network which can influence the degree of safety and electromagnetic compatibility of the electrical installation. Regulations pertaining to earthing systems differ largely among countries, though many follow international standards proposed by different international organisations, such as IEEE, IEC and BSI, particularly for low-voltage installations. Low-voltage systems employ voltages in the range 50-1000 V AC or 120–1500 V DC as defined by IEC [4]. Connection to earth constitutes a vital design objective in all electrical set-ups. One of the key essential requirements for designing an adequate earthing system is to have as low value as possible of resistance to remote earth in order to minimise the voltage between the earthing system and reference earth, known as Earth Potential Rise (EPR) which is proportional to the magnitude of the fault current and the earth resistance. The maximum permissible resistance value ranges from 1 Ω to 5 Ω for distribution substations depending on the local conditions [5]. Moreover, it is important to acknowledge that environmental factors surrounding the earthing system in any electrical installation also affect the performance of the installed earthing system [6]-[8]. Over the years, a considerable amount of research and development has been carried out to formulate accurate mathematical expressions of earth resistance for a wide range of earthing electrodes using empirical approximations. This has been carried out despite the small errors involved in practical prediction of the combined resistance of ground electrodes during site soil measurements while taking into account the multi-layer modelling uncertainties implicit of inherent resistivity inaccuracies. For complex earthing systems, a power system model in natural coordinates has been developed [9]. For simpler earthing systems, several formulae have been developed and published in the scientific literature which have considerably simplified the design calculations. The variety and wide choice of formulae presented in the international standards tend to confuse engineers, who might select different formulae, which are often ambiguously formulated, without critical examinations.

Most of the available earthing standards propose using a set of formulae and mathematical equations for calculating important parameters that aids in the design of a proper earthing system. Such parameters mainly include the earthing resistance and the minimum size of the earthing conductor, which can be calculated using various methods as part of the design methodology.

Lim, S. C. and Al-Shawesh, Y. [35] proposed a systematic step-by-step approach for the design of reliable and effective earthing systems for low-voltage installations. Preliminary layout design is to be conducted after deciding the type of earthing electrode.

The resistance of earthing must confirm to the minimum requirement. Further investigation of step and touch voltages is to be made to guarantee that they do not exceed the allowable limits. Nevertheless, if any of the three parameters still do not comply with standard regulations depending on the site soil conditions, alternative means of effective earthing methods must be considered in reducing the total earthing resistance and also the ground surface potential whenever design modification is necessary.

The main content of this research paper is organised in three sections. First, it comprehensively compiles all calculating design formulae categorised by the earthing approaches as prescribed in leading international standards and relevant works published by other researchers in the field of electrical ground earthing. Second, a quantitative comparison with analytical survey provides the calculation results in plots for all possible formulae studied for each earthing method employed in practice. Third, a preferred list of the most reliable mathematical formula, as identified from the survey, is proposed for each practical earthing configuration.

II. Earthing Types and Formulae

The simplest form of resistance of a typical hemisphere of earth can be calculated from the formula provided by the IEEE Green Book [1] as below:

$$R = \frac{\rho}{4\pi a} \quad (1)$$

This section collects many of the available formulae from popular earthing standards and also novel ones published in scientific articles for comparison of their differences, based on the arrangement and shape of the earthing electrodes as follows:

II.1. Plate

The minimum value of the resistance to earth in Ω afforded by a plate buried in soil of uniform resistivity may be calculated as proposed by BS-7430 [2] and IEEE-80 [5] from the following expression:

$$R_p = \frac{\rho}{4}\sqrt{\left(\frac{\pi}{A}\right)} \quad (2)$$

It is obvious that the resistance relies mainly on the soil resistivity and the area to be occupied, which are usually known in the early stages of the earthing design.

Laurent [10] and Niemann [11] proposed a formula which is also recommended by IEEE-80 [5] by adding a second term to Equation (2) for an upper limit of substation ground resistance, as follows:

$$R_p = \frac{\rho}{4}\sqrt{\left(\frac{\pi}{A}\right)} + \frac{\rho}{L} \quad (3)$$

For a buried horizontal round plate, the IEEE Green Book [1] proposes the following earthing resistance formula:

$$R_{hp} = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 - \frac{7}{12}\frac{a^2}{s^2} + \frac{33}{40}\frac{a^4}{s^4}\dots\right) \quad (4)$$

The IEEE Green Book [1] also suggests the following mathematical expression for the calculation of the grounding resistance of a buried vertical round plate:

$$R_{vp} = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 + \frac{7}{24}\frac{a^2}{s^2} + \frac{99}{320}\frac{a^4}{s^4}\dots\right) \quad (5)$$

II.2. Single Rod Electrode

The resistance of a grounding rod may be calculated as recommended by BS-7430 [2] from:

$$R_r = \frac{\rho}{2\pi L}\left[\log_e\left(\frac{8L}{d}\right) - 1\right] \quad (6)$$

Tagg [12] and Sunde [13] proposed a similar mathematical expression which is also presented in the IEEE Green Book [1] and ER/S34 [14] for the single rod electrode resistance as in Equation (6) by taking the rod radius instead of its diameter as follows:

$$R_r = \frac{\rho}{2\pi L}\left[\ln\left(\frac{4L}{a}\right) - 1\right] \quad (7)$$

As the diameter is twice the radius of the rod, both Equation (6) and Equation (7) obviously produce the same result of resistance value. Laurent [10] derived a different expression for calculation the earth resistance of a single rod electrode driven vertically as follows:

$$R_r = 0.366\frac{\rho}{L}\ln\left(\frac{3L}{d}\right) \quad (8)$$

Sullivan [15] introduced three types of rod model that were derived originally be Tagg [12] summarised as follows

Ellipsoid:

$$R_r = \frac{\rho}{2\pi r L}\ln\left(\frac{4L}{d}\right) \quad (9)$$

Uniform current:

$$R_r = \frac{\rho}{2\pi r L}\left[\ln\left(\frac{8L}{d}\right) - 1\right] \quad (10)$$

Cylinder:

$$R_r = \frac{\rho}{2\pi r L}\ln\left(\frac{2L}{d}\right) \quad (11)$$

Equation (10) for uniform current rod model is referred to in all standards and may be rewritten as:

$$R_r = \frac{\rho}{2\pi r L}\ln\left(\frac{8L}{ed}\right) \quad (12)$$

where r is the radius of an equivalent hemisphere in metres (m), e is 2.718, which divided into 8 gives 2.943.

Therefore, a simpler formula for a simplified uniform current model can expressed as follows:

$$R_r = \frac{\rho}{2\pi r L}\ln\left(\frac{3L}{d}\right) \quad (13)$$

Gomez [16] proposed calculating the earth resistance of a single vertical rod from Equation (8). On the contrary, he presented a new approximate expression for the length of rod for a vertical rod driven through a high resistivity superficial soil layer into a lower resistivity subsoil.

The adjustment was made by substituting an effective length of the earth rod $L' = L - h\,(1-(r_2/r_1))$, where L is the length of the rod in cm and h is the depth of the upper layer in cm. Thus, the new expression suggested by Gomez [16] becomes:

$$R_r' = 0.366\frac{r_1}{L'}\ln\left(\frac{3L'}{d}\right) \quad (14)$$

Laurent [10] derived Equation (8) based on the assumption that the rod electrode can be split up into small parts where the fault current flowing to earth is assumed distributed between these components. Then the potential at any point can be determined as the sum of all potentials arisen from each part. Tagg [12] focused on practical measurements of resistance of earth electrode and proposed Equation (7) based on the mathematical expressions originally derived by Dwight [17]. Sunde [13] independently formulated the same expression in Equation (7) for earth electrode resistance calculation based on the potential rise at the electrode midpoint. The longitudinal voltage drop along the rod electrode conductor can be neglected when finding the surface potential on the conductor. Thus, the difference in voltage across the surface of the conductor is assumed to be zero. ER/S34 [14] as one of the industry standards originally produced by the Electricity Association Engineering Recommendation in the UK included Equation (7) as proposed by Tagg [12] and Sunde [13].

Rudenburg [18] introduced another mathematical expression to be used for calculating the resistance of a vertically driven earth rod electrode as below:

$$R_v = \frac{\rho}{2\pi L}\left(\ln\left(\frac{2L}{a}\right) - 1\right) \tag{15}$$

Liew [19] proposed a new approximation for the resistance of a vertical grounding rod electrode when a low current with low frequency is injected into the electrode which is insufficient to initiate breakdown in the soil as given below:

$$R_o = \frac{\rho}{2\pi L}\left(\ln\left(\frac{L}{r_o}\right) + 1\right) \tag{16}$$

Zhang [20] proposed a simplified method for calculating the impulse resistance of vertically driven earthing rods by incorporating an effective radius of the grounding rod into the analytical formula of the low current resistance given in Equation (16). The equation of the impulse resistance is given by:

$$R_m = \frac{\rho}{2\pi L}\left(\ln\left(\frac{L}{r_e}\right) + 1\right) \tag{17}$$

where R_m is the impulse resistance of the vertical earth rod in Ω which is dependent on the impulse current injected into the rod, and r_e is the effective radius in metres (m) which varies with the current crest value.

Shukri et. al. [21] employed an equation that was originally proposed by BS-7430 [2] in their design by introducing slightly different parameters for vertical electrode in the following formula:

$$R_v = \frac{\rho}{4\pi L_r}\left[\ln\left(\frac{4L_r}{a}\right) - 1\right] \tag{18}$$

where L_r is the total length of the electrode in metres (m), and a is the nominal diameter of the vertical rod in metres (m), given that $L_r = NL_n$. Gomez [16] recommended calculating the earth resistance of a buried straight rod or wire from the following equation, provided that $h < 0.4\,L$:

$$R_s = \frac{0.366\rho}{L}\left(\ln\left(\frac{L}{d}\right) + \ln\left(\frac{L}{4h}\right) + 0.34\right) \tag{19}$$

II.3. Parallel Connection of Aligned Rods

Multiple interconnected rods can be utilised to reduce the earth resistance encountered in a single aligned rod.

Using a number of vertically driven rods, the minimum total resistance can be computed from the equation proposed by BS-7430 [2] as follows:

$$R_t = \frac{1}{n}\frac{\rho}{2\pi L}\left[\log_e\left(\frac{8L}{d}\right) - 1 + \frac{L}{s}\log_e\left(\frac{1.78n}{2.718}\right)\right] \tag{20}$$

Equation (20), carried out by Heppe R. J. [22] in 1998, is primarily based on the computational approach to the potential at the surface face of rods which offers a slightly more optimistic solution quantitively than is probably expected. This is due to the fact that this model is more closely aligned to the fundamental concept of electrostatic behaviour of an earthling system component, which obviously takes into account the interactive effect of the value of spacing (s). Considering this effect has long been practically realised in practice as being not actually less than twice the depth of the burial of the earthing rod. The reason behind this design concept is associated to the hemispherical radius of the earthing rod by avoiding utilising the constraint of less than two-times in system design process. If spacing (s) is reduced below the two-times value, this has an influence on the interference characteristics of multiple earthing electrode structures [2]. For two vertically driven ground rods where $s > L$, IEEE Green Book [1] suggested using the following equation:

$$R_{2r} = \frac{\rho}{4\pi L}\left(\ln\left(\frac{4L}{a}\right) - 1\right) + \frac{\rho}{4\pi s}\left(1 - \frac{L^2}{3s^2} + \frac{2}{5}\frac{L^4}{s^4}\cdots\right) \tag{21}$$

where $s < L$. IEEE Green Book [1] also proposed using the formula given below:

$$R_{2r} = \frac{\rho}{4\pi L}\left(\ln\left(\frac{4L}{a}\right) + \ln\left(\frac{4L}{s}\right) - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\cdots\right) \tag{22}$$

II.4. Vertical Strip or Round Conductor Electrodes

BS-7430 [2] recommended calculating the resistance of a single straight run of strip or round conductor with the use of the formula as follows:

$$R_s = \frac{\rho}{2\pi L} \log_e\left(\frac{L^2}{1.85hd}\right) \tag{23}$$

where d is the diameter of the round conductor or diameter of the equivalent cross-sectional area of the strip in metres (m). For two or more straight strips being connected together at one end and laid parallel to one another, the combined resistance may be determined from the equation proposed by BS-7430 [2] as follows:

$$R_n = FR_1 \tag{24}$$

Provided that $0.02 < \left(\frac{s}{L}\right) < 0.30.3$, F can be calculated from the three following equations for two, three, and four strips accordingly as recommended by BS-7430 [2].

For two lengths:

$$F = 0.5 + \left[0.078 \left(\frac{s}{L}\right)\right]^{-0.307} \tag{25}$$

For three lengths:

$$F = 0.33 + \left[0.071 \left(\frac{s}{L}\right)\right]^{-0.408} \tag{26}$$

For four lengths:

$$F = 0.25 + \left[0.067 \left(\frac{s}{L}\right)\right]^{-0.451} \tag{27}$$

II.5. Horizontal Strip or Round Conductor Electrode

For a horizontal strip electrode, BS-7430 [2] provided the following formula:

$$R_{hs} = \frac{\rho}{P\pi L}\left\{ \log_e\left(\frac{2L^2}{wh}\right) + Q\right\} \tag{28}$$

However, the IEEE Green Book [1] suggested a different mathematical expression to be used for computing the resistance of a buried horizontal strip as given below:

$$R_{hs} = \frac{\rho}{4\pi L}\left[\ln\left(\frac{4L}{a}\right) + \frac{a^2 - \pi ab}{2(a+b)^2} + \ln\left(\frac{4L}{s}\right) - 1 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512\,L^4}\cdots\right] \tag{29}$$

The strip used in Equation (29) has two sections a and b; section a should be more than section b by eight-times.

It means that, $b < a/8$. For a buried horizontal wire, the IEEE Green Book [1] proposed the following equation:

$$R_{hw} = \frac{\rho}{4\pi L}\left[\ln\left(\frac{4L}{a}\right) + \ln\left(\frac{4L}{s}\right) - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512\,L^4}\cdots\right] \tag{30}$$

Sunde [13] suggested another formula for a horizontal earth rod electrode given as follows:

$$R_h = \frac{\rho}{2\pi L}\left(\ln\left(\frac{2L}{\sqrt{2ha}}\right) - 1\right) \tag{31}$$

II.6. Ring of Wire Electrode

IEEE Green Book [1] suggested calculating the resistance of a ring of wire electrode from the formula below:

$$R_r = \frac{\rho}{2\pi^2 D}\left(\ln\left(\frac{8D}{d}\right) + \ln\left(\frac{4D}{s}\right)\right) \tag{32}$$

In order to calculate the steady state of a ring electrode, an equivalent formula recommended by Sunde [13] is as follows:

$$R_r = \frac{\rho}{2\pi^2 r}\left(\ln\left(\frac{8r}{\sqrt{2ad}}\right)\right) \tag{33}$$

where r is the outer radius of the ring in metres (m), a is the radius of the cross-sectional area of the ring in metres (m), and d is the depth of burial of the ring in metres (m).

Yasuda and Fujii [23] argued that Equation (33) would not be suitable for calculating the ring earth electrode resistance of an actual wind turbine because a grounding system of a wind turbine is composed of a ring earth electrode and a foundation for the turbine itself, which has a huge reinforcing concrete block. Using the numerical FDTD (Finite Difference Time Domain) method, Yasuda and Fujii [23] proposed a novel equivalent equation defined as follows:

$$R_r' = \frac{\rho}{2\pi^2 \alpha r_e}\left(\ln\left(\frac{8\beta r_e}{\sqrt{2ad}}\right)\right) \tag{34}$$

where r_e is the equivalent radius of the octagonal ring in metres (m), a is the radius of the cross-sectional area of the ring in metres (m), d is the depth of burial of the ring in metres (m), and the improvement constants: $\alpha = 1.85$ and $\beta=3.82$ determined according to the least square method.

Gomez [16] recommended calculating the earth resistance of a buried circle of wire from this formula:

$$R_c = \frac{0.366\,r}{L}\left(\ln\left(\frac{L}{d}\right) + \ln\left(\frac{L}{4h}\right) + 0.81\right) \tag{35}$$

where r is the soil resistivity in (Ω cm), h is the depth of burial in (cm), L is the total length of the circle wire in (cm), and d is the diameter of wire in (cm).

II.7. Mesh (Grid)

The minimum resistance value of a mesh (grid) may be calculated from the following equation as proposed by BS-7430 [2]:

$$R_m = 0.443\frac{\rho}{\sqrt{A}} + \frac{\rho}{L} \quad (36)$$

where A is the actual area covered by the mesh in square metres (m^2) and L is the total length of strip used in the mesh in metres (m). Equation (2) is also applied to square-shaped grids in substations as proposed by IEEE [5]. The upper limit to the earthing resistance of these grids can be obtained by using the formula given by Laurent [10] and Niemann [11] which is stated above for plate earthing in Equation (3). The simplest formula for the earth resistance of a grid proposed by S34 [24] and referred to by EA 41-24 [25] is given as:

$$R_g = \rho\left(\frac{1}{4r} + \frac{1}{L}\right) \quad (37)$$

where r is the radius of a thin circular plate in metres (m) and L is the combined length of conductor forming a grid in metres (m). IEEE-80 [5] and BS-7430 [2] give another formula as:

$$R_g = \rho\left\{\frac{1 + \frac{r}{r + 2.5h}}{8r} + \frac{1}{L}\right\} \quad (38)$$

where r is the radius of a thin circular plate in metres (m) and L is the combined length of conductor forming a grid in metres (m). Equation (3) consists the second term $\frac{\rho}{L}$ accounting for the fact that resistance of any earthing system composed of a finite number of conductors is always higher than the resistance of a solid metallic plate.

As the total length of conductors gets infinitely large, the difference in resistance reduces and approaches zero.

Guemes-Alonso [26] proposed calculation the ground resistance of earthing grids with square meshes buried in uniform soils using the FEM (Finite Element Method) from the following equation:

$$R_g = \frac{0.347\,\rho}{L^{0.414}\,W^{0.517}\,A^{0.43}} \quad (39)$$

where L is the number of meshes in direction X, and W is the number of meshes in direction Y. While Expression (2) and Expression (3) may be utilised with reasonable accuracy for square-shaped grids buried in the ground soil at depths of less than 0.25 metres, for grids buried between a minimum depth of 0.25 metres and a maximum depth of 2.5 metres, Sverak [27] introduced a correction factor considering the effect of variation in the burial depth in the following expression:

$$R_g = \rho\left[\left(\frac{1}{L_T}\right) + \frac{1}{\sqrt{20A}} + \left(1 + \frac{1}{1 + h\sqrt{\frac{20}{A}}}\right)\right] \quad (40)$$

where A is the actual area covered by the grid in square metres (m^2). Schwarz [28] proposed an expression by using equations introduced by Sunde [13] and Rudenburg [18] for grids without ground rods embedded in a soil of uniform resistivity as given below:

$$R_g = \left(\frac{\rho}{\pi L_c}\right)\left[\ln\left(\frac{2L_c}{a'}\right) + \frac{k_1 L_c}{\sqrt{A}} - k_2\right] \quad (41)$$

where A is the actual area covered by conductors in square metres (m^2), and a' is $\sqrt{a2h}$ for conductors buried at depth h in metres (m). A limitation of the above Equation (41) is the coefficients k_1 and k_2 that can be attained by using graphs only, which makes an analytical solution unattainable with the use of a computer.

However, this deficiency was overcome by Kercel's [29] expressions for k_1 and k_2 given as shown below:

$$k_1 = 1.84\sqrt{\frac{ab}{2}}\left\{\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 + b^2}}{b}\right) + \right.$$
$$+ \frac{1}{b}\ln\left(\frac{b + \sqrt{a^2 + b^2}}{a}\right) + \frac{a}{3b^2} + \quad (42)$$
$$\left. + \frac{b}{3a^2} - \left(\frac{(a^2 + b^2)^{\frac{3}{2}}}{3a^2b^2}\right)\right\}$$

$$k_2 = \ln\left(\frac{4(a + b)}{b}\right) + \frac{2k_1(a + b)}{\sqrt{ab}} +$$
$$- \ln\left(\frac{a + \sqrt{a^2 + \left(\frac{b}{2}\right)^2}}{\left(\frac{b}{2}\right)}\right) + \quad (43)$$
$$- \frac{1}{2}\ln\left(\frac{\left(\frac{b}{2}\right) + \sqrt{a^2 + \left(\frac{b}{2}\right)^2}}{-\left(\frac{b}{2}\right) + \sqrt{a^2 + \left(\frac{b}{2}\right)^2}}\right)$$

where b is the length of the long side of the grid and a is the length of the short side of the grid in metres (m).

II.8. Concrete-Encased Electrode

Concrete is hygroscopic with low resistivity as it attracts moisture and thus behaves as a semiconducting medium [6]. A concrete buried in soil has a typical resistivity of 30-90 Ω m. This particular medium can be used in highly resistive soils where a metallic rod or wire is encased in concrete which lower the electrode resistance than a similar electrode buried directly in ground earth because this encasement lessens the resistivity of the material surrounding the metal element.

However, there are two disadvantages of this method.

On the one hand, corrosion of the rebar material can be caused in case where a small DC current is present.

On the other hand, the concrete may be broken due to pressure produced by corroded steel or when large quantity of current passes through the concrete which can vaporise its moisture. Concrete-encased metal rod constituted a major breakthrough in earthing systems technology due to its effective performance and now it is one of the best alternatives to pipe and driven rod electrodes buried in the soil. Wiener [30] showed that electrodes that are encased in concrete are capable of withstanding very large amount of earth current, and the rate of corrosion of such electrodes is lower than that of electrodes driven directly in ground earth. The resistance of a backfilled electrode encased in low resistivity material such as a conducting concrete can be calculated from the following equation as proposed by BS-7430 [2]:

$$R_b = \frac{1}{2\pi L}\left[(\rho - \rho_c)\left(\log_e\left(\frac{8L}{d}\right) - 1\right) + + \rho_c\left(\log_e\left(\frac{8L}{d}\right) - 1\right)\right] \quad (44)$$

Fagan and Lee [31] established a new formula to calculate the earth resistance of a vertically driven rod encased in concrete given as below:

$$R_{CE-rod} = \frac{1}{2\pi L_r}\left[\rho_c\left(\ln\left(\frac{D_c}{d}\right)\right) + \rho\left(\ln\left(\frac{8L_r}{D_c}\right) - 1\right)\right] \quad (45)$$

Equation (45) can be related to the commonly used Expression (6) for a single earth rod electrode as proposed by BS-7430 [2] as given by:

$$R_r = \frac{\rho}{2\pi L}\left[\log_e\left(\frac{8L}{d}\right) - 1\right] \quad (46)$$

Then Equation (45) can be resolved into the following as presented by Fagan and Lee [31]:

$$R_{CE-rod} = \frac{1}{2\pi L_r}\left\{\rho\left[\ln\left(\frac{8L_r}{D_c}\right) - 1\right] + + \rho_c\left[\ln\left(\frac{8L_r}{d}\right) - 1\right] + - \rho_c\left[\ln\left(\frac{8L_r}{D_c}\right) - 1\right]\right\} \quad (47)$$

The latter term of Equation (47) is obtained hypothetically as a difference of resistance values of a rod concrete, for the case where d and D_c are entered into Equation (6) of the single-medium, ρ is replaced by ρ_c.

It should be noted that Equation (47) represents a combination of two resistances in series namely the earthing resistance formed by Equation (6) of a concrete cylinder of diameter D_c buried directly in the soil of resistivity ρ, and the earthing resistance of the inner segment of diameter D_c which contains the metal rod of diameter d. Siow et.al [32]-[34] has also proposed an improvised concrete material by introducing bentonite into the mixture to lower its resistivity. It was found that for a specific dimension, the earthing resistance is only a factor of 0.3 of the localized soil resistivity.

II.9. Miscellaneous Electrodes

There are many configurations of rod electrodes, but a few of those have been proposed in the standards which the earthing system designer is most likely to attempt first in order to obtain the required resistance value, particularly when dealing with deep reinforced piles.

II.9.1. Three Rods at the Vertices of an Equilateral Triangle

The resistance of three interconnected rods set out at the vertices of an equilateral triangle could be determined from the formula proposed by BS-7430 [2] as follows:

$$R_e = \frac{1}{3}\left[2\left(\log_e\left(\frac{8L}{d}\right) - 1\right) - 1 + 2Ls\right] \quad (48)$$

where s is the length of one side of the equilateral triangle in m.

II.9.2. Right-Angle Turn of Electrode

For two strips of an equal length set at 90° to each other meeting at one corner, the resistance can be computed from the following equation as suggested by BS-7430 [2]:

$$R_L = \frac{\rho}{2\pi L}\log_e\left(\frac{L^2}{1.27hd}\right) \quad (49)$$

where L is the total length of strip in m and d is the diameter of the round conductor or diameter of the equivalent cross-sectional area of the strip in m.

The IEEE Green Book [1] proposed another expression for a right-angle turn of wire given as below:

$$R_L = \frac{\rho}{4\pi L}\left(\ln\left(\frac{2L}{a}\right) + \ln\left(\frac{2L}{s}\right) - 0.2373 + +0.2146\frac{s}{L} + 0.1035\frac{s^2}{L^2} - 0.0424\frac{s^4}{L^4} \ldots\right) \quad (50)$$

where L is the length of the arm in metres (m).

II.9.3. Three-Point Star

BS-7430 [2] proposed calculating the resistance of three strips set at 120° meeting at the star point all of equal length from the formula given below:

$$R_{3s} = \frac{\rho}{2\pi L}\log_e\left(\frac{L^2}{0.767hd}\right) \quad (51)$$

where L is the total length of strip in metres (m) and d is the diameter of the round conductor or diameter of the equivalent cross-sectional area of the strip in metres (m).

IEEE Green Book [1], suggested another equation to be used for calculating the earth resistance of a three-point star electrode as follows:

$$R_{3s} = \frac{\rho}{6\pi L}\left(\ln\left(\frac{2L}{a}\right) + \ln\left(\frac{2L}{s}\right) + 1.071 + -0.209\frac{s}{L} + 0.238\frac{s^2}{L^2} - 0.054\frac{s^4}{L^4} \ldots\right) \quad (52)$$

where L is the total length of arm strip in metres (m) and a is the radius of the round conductor or the radius of the equivalent cross-sectional area of the strip in metres (m).

II.9.4. Four-Point Star

For four strips set out in a cruciform, the resistance can be calculated using the expression suggested by BS-7430 [2] as given below:

$$R_{4s} = \frac{\rho}{2\pi L}\log_e\left(\frac{L^2}{0.21767hd}\right) \tag{53}$$

where L is the total length of strip in m and d is the diameter of the round conductor or diameter of the equivalent cross-sectional area of the strip in m.

The IEEE Green Book [1] recommended using the below formula when calculating the resistance of a four-point star electrode:

$$R_{4s} = \frac{\rho}{8\pi L}\left(\ln\left(\frac{2L}{a}\right) + \ln\left(\frac{2L}{s}\right) + 2.912 -1.071\frac{s}{L} + 0.645\frac{s^2}{L^2} - 0.145\frac{s^4}{L^4}\ldots\right) \tag{54}$$

where L is the total length of arm strip in m and a is the radius of the round conductor or the radius of the equivalent cross-sectional area of the strip in m.

II.9.5. Six-Point Star

The resistance of a six-point star earth electrode can be calculated using the formula proposed by the IEEE Green Book [1] as shown below:

$$R_{6s} = \frac{\rho}{12\pi L}\left(\ln\left(\frac{2L}{a}\right) + \ln\left(\frac{2L}{s}\right) + 6.851 + -3.128\frac{s}{L} + 1.758\frac{s^2}{L^2} - 0.490\frac{s^4}{L^4}\ldots\right) \tag{55}$$

where L is the total length of arm strip in metres (m) and a is the radius of the round conductor or the radius of the equivalent cross-sectional area of the strip in metres (m).

II.9.6. Eight-Point Star

The resistance of the earthing electrode arranged in eight-point star can be calculated using the mathematical expression suggested by the IEEE Green Book [1] given as below:

$$R_{8s} = \frac{\rho}{16\pi L}\left(\ln\left(\frac{2L}{a}\right) + \ln\left(\frac{2L}{s}\right) + 10.98 + -5.51\frac{s}{L} + 3.26\frac{s^2}{L^2} - 1.17\frac{s^4}{L^4}\ldots\right) \tag{56}$$

where L is the total length of arm strip in m and a is the radius of the round conductor or the radius of the equivalent cross-sectional area of the strip in m.

II.9.7. Structural Concrete-Encased Steelwork

The underground metalwork of a concrete foundation may also be utilised as an effective ready-made electrode for the earthing system.

The total area of the electrode formed by the metalwork of a large structure is commonly used because it furnishes a lower earth resistance than that attainable by other earthing methods.

It is possible to obtain an overall value below 1 Ω using this method. However, corrosion of the metalwork reinforcement might occur where corrosion products dominate a greater volume than the existing metal and thus most likely cause cracking. Therefore, it is highly important to take this into consideration especially the continuous earth currents since such currents from a possible source might be incompatible with other metalwork, comprising other earth electrode types to which the foundation buried metalwork may be bonded. In this case, a cathodic protection might be necessary to employ [2]. Arcing or the rapid moisture evaporation might occur causing damage to the foundation concrete whereby cracking due to long durations of earth fault current exceeding the electrode carrying capability.

There are three factors that determines the earth resistance of a concrete-encased steelwork or of reinforced concrete bars:

a) Soil type;
b) Moisture content of the soil;
c) Foundation design.

Low resistivity of approximately 30 Ω m to 90 Ω m can be achieved by concrete at normal temperatures as concrete is hygroscopic except in dry soil [2].

Combined resistance of all earth electrodes should be achieved. For a structure covering a large area, the earth resistance might be fairly low and thus it might be difficult to obtain accurate measurements on a complete structure.

However, it may be more satisfactory to measure the resistance of one footing before it is connected to other footings electrically and before there are any other paralleled footings nearby in a structure of similar footings.

Measurement of several such footings is highly recommended to gain an indication of their resistance variation. Assuming the resistance of one single footing as a typical value, the combined effect of all similar footings $\{R_{TOT}\}$ arranged in a rectangular plan can be calculated from the formula proposed by BS-7430 [2] given as below:

$$R_{TOT} = \frac{R_1}{n}\left(1 + \left(\frac{\lambda\rho}{2\pi R_1 s}\right)\right) \tag{57}$$

The spacing should be adjusted so that the ratio $\left(\frac{\rho}{2\pi R_1 s}\right)$ is less than 0.2. Since the concrete to earth is dependent on its moisture content, allowance should be made for any increase in the resistance of the electrode as the largest portion of resistance is immediately around the metalwork.

In case where a contact between metalwork within the concrete or below ground such as reinforcing bars, welding above ground is recommended at anchor bolts by attaching a bond conductor to bypass each structural joint.

This will ensure the electrical continuity of all metalwork installed as part of the electrode [2]. The effect of using multiple concrete-encased rebar earthing electrodes arranged in a hollow rectangular configuration may be estimated by using the method of Tagg [12] of equivalent hemispherical electrodes illustrated in the following equation which is also proposed by the S34 Standard [24]:

$$M = \frac{\text{resistance of } N \text{ electrodes in parallel}}{\text{resistance of 1 electrode}} = \frac{1 + k\alpha}{N} \quad (58)$$

$$R_a = \frac{R_1}{n}(1 + k\alpha) \quad (59)$$

where $\alpha = r/d$, r is the radius of equivalent hemisphere which is 0.762 m for 10 foot (3.048 m) electrode ≥ 6-inch diameter, d is the spacing of electrodes in feet, and M is multiplier to obtain array resistance from single electrode resistance. Values of k and M can only be obtained from graphs.

However, for practical purposes, $M = 2/N$ when $N \geq 10$.

An expression for k was introduced by Sullivan [15] which is acceptable within reading errors at four and eight rods (9 % and 6 % high, respectively), $k = 2.16 \ln(n)$. Using the k equation, Equation (59) can be rewritten as:

$$R_a = \frac{\rho}{2\pi r n}[1 + 2.16 \ln(n)\, \alpha] \quad (60)$$

Equation (60) is valid for rectangles and a square as the resistance R_a is dependent on the rod spacing which increases for a given area as the side lengths increase.

The array resistance can be obtained using the perimeter length P divided by the number of rods n, Equation (60) then becomes:

$$R_a = \frac{\rho}{2\pi r n}\left[1 + \frac{2.16\, r n \ln(n)}{P}\right] \quad (61)$$

Tagg [12] mentioned that adding multiple electrodes to fill in a hollow square configuration is not useful in reducing the earthing resistance considerably. For a reinforced concrete foundation where only vertical reinforcing rods are bonded to the structure of the building or to the earthing system, the resistance to earth can be calculated using the equation proposed by BS-7430 [2] given as follows:

$$R_r = \frac{1}{2\pi L}\left[(\rho_c - \rho)\log_e\left(1 + \frac{\delta}{z}\right) + \rho \log_e\left(\frac{2L}{z}\right)\right] \quad (62)$$

where R_r is the resistance of the reinforced concrete foundation in Ω, L is the length of reinforcing rod below ground level in m and z is the geometric mean distance of rod cluster in m which can be calculated from Table I.

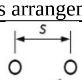
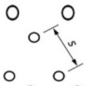
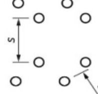
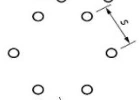
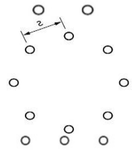
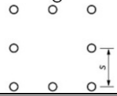
III. Case Study Calculations and Analysis

As the earthing resistance depends primarily on the resistivity of the soil regardless of the earthing type or configuration, there is a proportional relationship between the two parameters. If the soil resistivity is high, then the earthing resistance will be high and vice versa. In this research work, the comparison of the various formulae of earthing resistance is carried out for selected different values of soil resistivity that ranges between 10 Ω m and 500 Ω m. The range of soil resistivity values was selected based on practical measurements for a typical natural soil. The assumption of values of electrodes sizes taken for this case study survey are of the most common standard nominal values obtainable in the market according to manufacturing datasheets used. This allows us to assess the effects of differences of the various formulae. The corresponding earthing resistances investigated for each formula are plotted in line graph as shown for each earthing configuration in Figures 1-10.

Since Lim, S. C. and Al-Shawesh, Y. [35]-[36], newly developed a comprehensive, economically viable and technically acceptable design template which includes all the equations studied in this case study survey, this accessible software tool was used for automatic, effortless and instant computations of these parameters as proven useful for industry engineers designing effective earthing systems.

Then, manual calculations were done to verify all the results obtained and validate the precision and accuracy of computations.

TABLE I
GEOMETRIC MEAN DISTANCE Z FOR CLOSELY SPACED REINFORCING RODS IN A SYMMETRICAL PATTERN [2]

Number of rods	Rods arrangement	z value (m)
2		$\sqrt[2]{as}$
3		$\sqrt[3]{as^2}$
4		$\sqrt[4]{2as^3}$
6		$\sqrt[6]{6as^5}$
8		$\sqrt[8]{52as^7}$
8		$\sqrt[8]{23as^7}$

III.1. Plate

The earthing resistance values versus soil resistivity is plotted in Figure 1 for Equation (2), Equation (3), Equation (4), and Equation (5) which were contrasted by substituting the values of parameters chosen as practical values as described in Table II below. As can be seen from Figure 1, Equation (3) is the safest formula compared to other formulae for the same plate earthing configuration as it gives the highest resistance value.

This formula was proposed by Laurent [10] and Niemann [11] and also recommended by IEEE-80 [5] by considering the upper limit of the resistance including the length of buried conductor in the equation. Moreover, Equation (2) proposed by BS-7430 [2] is more preferred in comparison to the other two formulae suggested by IEEE Green Book [1].

III.2. Single Rod Electrode

Equation (8) to Equation (23) were compared with various soil resistivities plotted in Figure 2 with a model single rod electrode having the parameters as given below in Table II. As shown in Figure 2, Equation (14) is the most conservative formula for single rod electrode earthing which is proposed by Gomez [16] by considering the resistivity of upper layer and resistivity of subsoil.

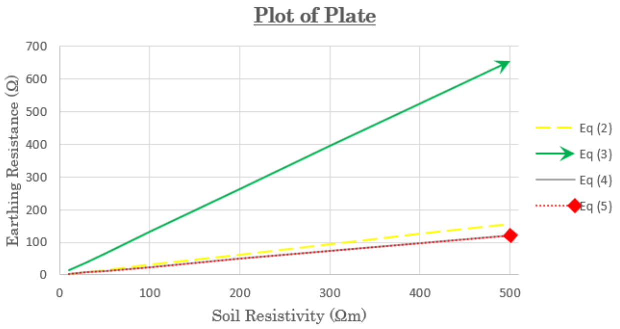


Fig. 1. Plot of Plate

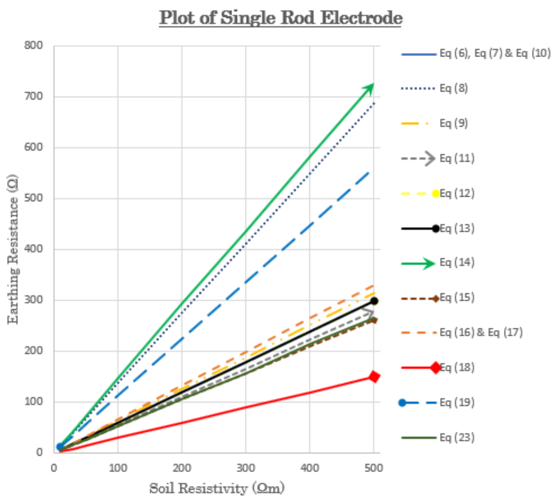


Fig. 2. Plot of Single Rod Electrode

However, Gomez [16] does not clarify how Equation (14) was derived nor does he mention for what case, thus a more acceptable expression for a vertically driven rod resistance can be selected from Equation (8) proposed by Laurent [10].

III.3. Parallel Aligned Rods

Assuming two aligned rods connected in parallel where each rod = 1.5 m long, and each rod = 16 mm in diameter, the resistance versus resistivity values for parallel aligned rods are plotted in Figure 3. Based on the results shown on Figure 3, the safest formula for earthing resistance is Equation (22) proposed by IEEE Green Book [1], where the spacing between two aligned rods is less than the length of each rod. However, this equation is only applicable for two parallel electrodes. For more than two rods, the only available equation is as proposed by BS-7430 [2] as given in Equation (20).

III.4. Horizontal Strip/Round Conductor

The resistances versus the soil resistivities for Equation (28) to Equation (31) are plotted in Figure 4 by assuming the values of parameters shown in Table II. As can be noted from Figure 4, Equation (28) is the most conservative formula as proposed by BS-7430 [2].

III.5. Ring of Wire

Equation (32) to Equation (35) are compared via a plot of the earthing resistance versus the soil resistivity in Figure 5 by substituting the values of parameters in Table II.

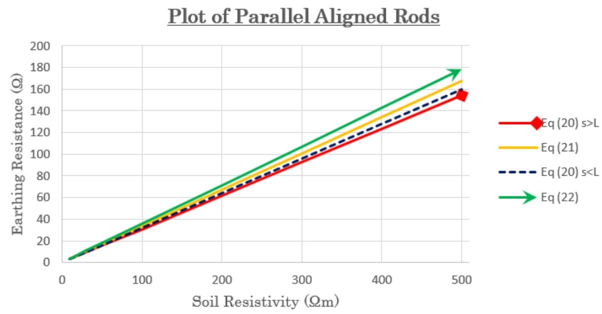


Fig. 3. Plot of Parallel Aligned Rods

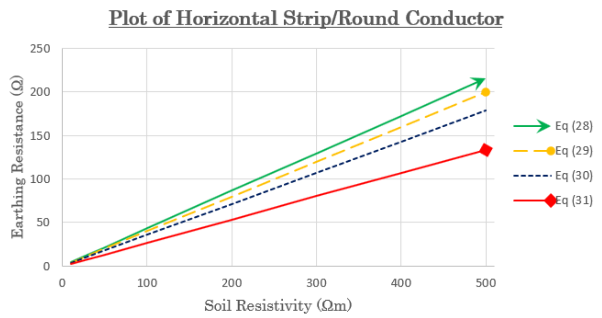


Fig. 4. Plot of Horizontal Strip/Round Conductor

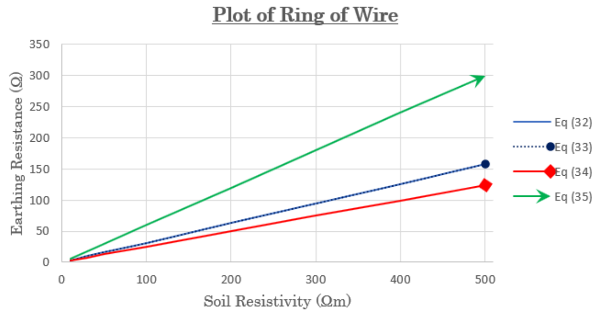


Fig. 5. Plot of Ring of Wire

By referring to the results presented in Figure 5, Equation (35) is the most conservative formula, which is recommended by Gomez [16].

III.6. Mesh (Grid)

Equation (2), Equation (3), and Equation (36) to Equation (41) are contrasted in graphic presentations of earthing resistance versus soil resistivity as shown in Figure 6. The parameters of all formulae were assumed to have the values shown in Table II. From Figure 6, it can be noted that Equation (40) provides the most conservative value for earthing resistance which is proposed by Sverak [27].

III.7. Concrete-Encased Electrode

Equation (44) to Equation (47) are compared by plotting resistance versus soil resistivity as demonstrated in Figure 7. The values of parameters are substituted identically in each equation as given in Table II.

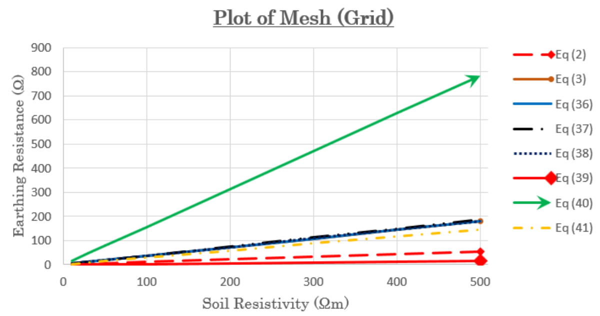


Fig. 6. Plot of Mesh (Grid)

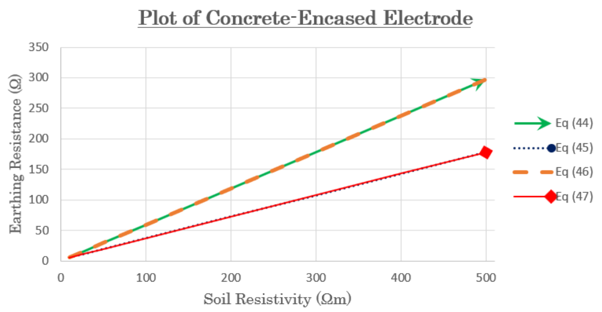


Fig. 7. Concrete-Encased Electrode

As shown in Figure 7, both Equation (44) and Equation (46) produce the same results and they can be taken as the most conservative formulae to be used for concrete-encased electrode as proposed by BS-7430 [2].

III.8. Right-Angled Turn of Electrode

By plugging the values of parameters given in Table II in Equation (49) and Equation (50), the earthing resistances for different soil resistivities are plotted in Figure 8. As can be noted from Figure 8, Equation (49) is the most conservative formula as recommended by BS-7430 [2].

III.9. Three-Point Star

In order to compare Equation (51) and Equation (52), identical values of parameters were plugged into the equations as presented in Table II. Figure 9 shows the line graph of the resistances versus soil resistivity for both formulae.

As can be seen from Figure 9, Equation (51) gives the most conservative calculation for earthing resistance as proposed by BS-7430 [2].

III.10. Four-Point Star

Similarly, a comparison of Equation (53) and Equation (54) is demonstrated via the line graph of resistances for various soil resistivities as shown in Figure 10 by assuming the parameters provided in Table II.

Equation (53) as proposed by BS-7430 [2] is the safest formula for earthing resistance calculation.

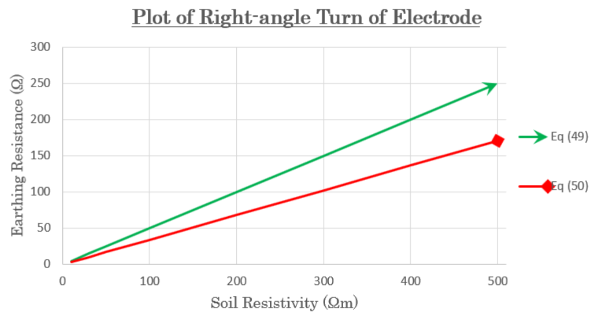


Fig. 8. Plot of Right-Angle Turn of Electrode

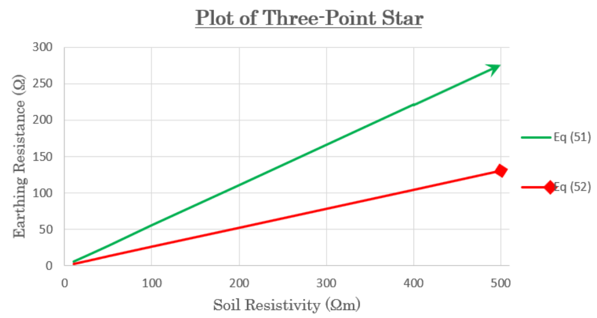


Fig. 9. Plot of Three-Point Star

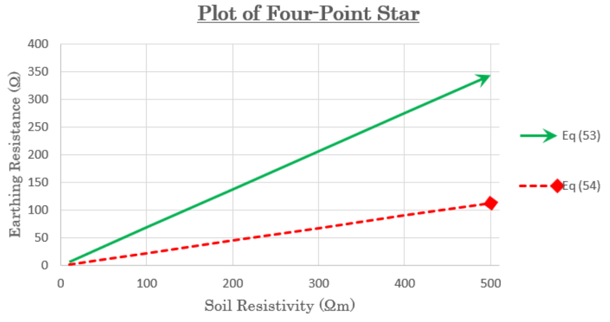


Fig. 10. Plot of Four-Point Star

TABLE II
VALUES OF PARAMETERS SUBSTITUTED IN THE FORMULAE FOR EACH EARTHING CONFIGURATION

Earthing Type	Values of Parameters
Plate Earthing	Area, A = 2 m^2, Depth of Burial = 1.8 m, Spacing = 3.6 m, Radius of Plate, a = 0.564 m
Single Rod Electrode	Length of Rod = 1.5 m, Diameter of Rod = 0.016 m, Radius of Rod = 0.008 m, Radius of Equivalent Hemisphere = 1 m
Parallel Aligned Rods	Length of each rod = 1.5 m, Diameter of each rod = 16 mm. When $s > L$, Spacing, s = 2 m, When $s < L$, Spacing, s = 1 m
Horizontal Strip/Round Conductor	Width = 16 mm, Length, L = 1.5 m, P = 4, Q = 1.8, Radius, a = 16 mm, b = 0.0001
Ring of Wire	D = 1 m, Diameter of Wire = 0.016 m, s = 4 m, Radius of Wire = 0.008 m, Depth = 2 m, Outer Radius = 0.5 m, β = 3.82, α = 1.8. Radius of Octagon Ring = 0.48684 m
Mesh (Grid)	L = 4 m, A = 16 m^2, r = 2 m, L_t = 4 m, h = 2.5 m, a = 0.008 m, k_1 = 4, k_2 = 2
Concrete-Encased Electrode	L = 1.5 m, ρ_c = 10 Ωm, D_c = 0.16 m, d = 0.016 m
Right-angle Turn of Electrode	A = 8 mm, d = 16 mm, L = 1.5 m, Depth = 1 m
Three-point Star	A = 8 mm, d = 16mm, L = 1.5 m, Depth = 1 m
Four-point Star	a = 8 mm, d = 16 mm, L = 1.5 m, Depth = 1 m

IV. List of Preferred Formulae

Based on the analytical results obtained from the comparative review outlined in Section III, it is evident that there are varying trends for the formulae used to optimise the earthing system design. This is an indication of the empirical nature in deriving these equations which includes a magnitude of errors to varying degrees as compared with the relevant formulae in the international standards considered in this case study. While precision is not always possible in mathematical derivations, even with an accurate soil resistivity measurement, the nonuniformity of ground soil layers has to be taken into account.

Most importantly, a systematic selection of proper equations with adequate accuracy for the design calculations is imperative in order to avoid the serious errors involved in using conventional formulae. Table III below summarises, in a proposed list, a set of the preferred and most conservative formulae, from a safety perspective, for each earthing configuration and method.

TABLE III
SUMMARY OF THE MOST CONSERVATIVE FORMULA FOR EACH EARTHING CONFIGURATION

Earthing Type	Formula
Plate Earthing	$R_p = \frac{\rho}{4}\sqrt{\left(\frac{\pi}{A}\right)} + \frac{\rho}{L}$
Single Rod Electrode	$R_r' = 0.366\frac{r_1}{L'}\ \ln\left(\frac{3L'}{d}\right)$
Parallel Aligned Rods	$R_{2r} = \frac{\rho}{4\pi L}\left(\ln\left(\frac{4L}{a}\right) + \ln\left(\frac{4L}{s}\right) - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4}\dots\right)$
Horizontal Strip/Round Conductor	$R_{hs} = \frac{\rho}{P\pi L}\left\{\log_e\left(\frac{2L^2}{wh}\right) + Q\right\}$
Ring of Wire	$R_c = \frac{0.366\,r}{L}\left(\ln\left(\frac{L}{d}\right) + \ln\left(\frac{L}{4h}\right) + 0.81\right)$
Mesh (Grid)	$R_g = \rho\left[\left(\frac{1}{L_T}\right) + \frac{1}{\sqrt{20A}} + \left(1 + \frac{1}{1 + h\sqrt{\frac{20}{A}}}\right)\right]$
Concrete-Encased Electrode	$R_r = \frac{\rho}{2\pi L}\left[\log_e\left(\frac{8L}{d}\right) - 1\right]$
Right-angle Turn of Electrode	$R_L = \frac{\rho}{2\pi L}\log_e\left(\frac{L^2}{1.27hd}\right)$
Three-point Star	$R_{3s} = \frac{\rho}{2\pi L}\log_e\left(\frac{L^2}{0.767hd}\right)$
Four-point Star	$R_{4s} = \frac{\rho}{2\pi L}\log_e\left(\frac{L^2}{0.21767hd}\right)$

V. Conclusion

The design of an earthing system is quite complex and requires meticulous and precise calculations for parameters to meet national and international standards.

The most essential criteria for an adequate and a reliable earthing system is that the earthing resistance must ensure the lowest value to earth enough to dissipate earth fault currents. The variety of choice for formulae tends to make the decision by the designer uncertain and ambiguous. This is due to absence of clarification and agreement of universal standards, especially with regards to geographical locations where different international standards are required simultaneously, such as in Malaysia. This paper collected all earthing techniques and their formulae from widely used global standards and other proposed methods available in published articles to investigate the effects of their differences. A comparative analytical study of the various mathematical expressions used to calculate the earthing resistance was described in plots based on the earthing type and electrode configuration. The most conservative formulae were identified for all earthing types where comparison was applicable. This was accomplished by noting the trends of equations which yielded the highest values of resistance compared to those computed with other formulae. All the results displayed in the line graphs were tested by manual calculations and verified for high accuracy and confirmed to be error-free. Based on a case study to find the deficiencies and differences of those formulae for various soil resistivities, a proposed list of preferred formulae for different earthing techniques was presented as the most suitable mathematical expressions for earthing design calculations for all types of earthing structures. This list is thought to be of great practicality, useful in the design of ground earthing for low-voltage installations by taking into consideration the highest integrity and maximum safety of the overall electrical earthing system. It can be concluded that among the standards, BS-7430 [2] proposed safer formulae as compared to the IEEE Green Book [1]. Among the published papers, Gomez [16] introduced two of the most conservative expressions for earthing resistance calculations compared to any other formula studied for the specified layouts. The results presented in this article are of great utility, as a reference for the organisations concerned in setting international standardisation for electrical safety as well as provisioning universal standards for earthing systems.

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